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ACOUSTIC WAVE INTERACTION WITH BODIES COVERED BY A THIN COMPRESSIBLE LAYER*

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The problem of acoustic wave interaction with rigid bodies on whose surface there is a thin compressible layer is formulated. The motion of the material is assumed to be quasi-two-dimensional in the layer, which results in a problem with special boundary condition, which generalizes the problem of acoustic wave diffraction by a rigid body and a cavity. The problem of plane acoustic wave diffraction by a sphere covered with a thin compressible layer is solved.

1. Formulation of the Problem.

Let curvilinear orthogonal coordinates in space define the radius-vector of the point $\mathbf{r}(q_1, q_2, q_3)$. A thin layer of an ideal compressible fluid of variable thickness, whose outer surface is $\mathbf{r}=\mathbf{r}(q_1, q_2, a+h)$ where h is a function of q_1, q_2 and time t, is attached to the surface of the rigid body described by the parametric equation $\mathbf{r}=\mathbf{r}(q_1, q_2, a)$, where a is a constant. The space outside the body and the compressible layer is filled with an ideal fluid with physical characteristics different from the characteristics of the layer material on the body. The reflection of a pressure wave from the body is investigated later. The problem under consideration arises when studying the diffraction by bodies with thin damping coatings that are in water when there are gas bubbles on their surface, and in other cases.

In general, when a pressure wave acts on a body covered by a compressible layer, complex three-dimensional fluid flow occurs in the layer. Because of the thinness of the layer, it is natural to try to reduce the problem of the flow in a layer to a quasi-two-dimensional flow over a surface $\mathbf{r}=\mathbf{r}(q_1, q_2, a)$ /1/. Let us formulate the constraints on the conditions of the problem under which this can be done successfully. First, because of the thinness of the layer, the Lamé parameters $H_1=|\partial\mathbf{r}/\partial q_1|$ and $H_2=|\partial\mathbf{r}/\partial q_2|$ can be assumed to be independent of the coordinate q_3 for $a\leqslant q_3\leqslant a+h$. It can be shown that this assumption will be satisfied with sufficient accuracy, when appropriate derivatives of $\mathbf{r}(q_1, q_2, q_3)$ exist, if the following inequalities are satisfied

$$\left|\frac{\partial H_i(q_1, q_2, a)}{\partial q_3}\right| \frac{h(q_1, q_2, t)}{H_i(q_1, q_2, a)} \ll 1, \quad i=1,2 \tag{1.1}$$

To simplify the calculations, the coordinate q_3 is identified with the arc-length of the appropriate coordinate line, i.e., it is assumed that $H_3=|\partial\mathbf{r}/\partial q_3|=1$, as can always be done by an appropriate selection of the coordinate system.

We will further assume that the pressure p in the layer and the density ρ are independent of the coordinate q_3. Introducing this assumption, we will neglect the waves in the direction of the normal to the surface $\mathbf{r}=\mathbf{r}(q_1, q_2, a)$. For the one-dimensional case, the validity of this assumption of proved in /1/ in an acoustic formulation for a small value of the ratio between the acoustic impedance of the layer material and the acoustic impedance of the surrounding fluid. Moreover, it is assumed, for simplicity, that the flows in the layer and in the surrounding fluid are barotropic.

Let v_1, v_2, v_3 be components of the fluid velocity vector in the layer. It is assumed that the components v_1 and v_2 depend slightly on the coordinate q_3, and we also consider their mean values

$$\langle v_i\rangle = \frac{1}{h}\int_a^{a+h} v_i\,dq_3 \quad (i=1,2)$$

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$$\langle v_i^2\rangle = \langle v_i\rangle^2 \ (i = 1,\ 2),\ \langle v_1v_2\rangle = \langle v_1\rangle\langle v_2\rangle \tag{1.2}$$

which are natural for slowly varying functions.

The assumptions regarding the functions p, ρ, v_1, v_2 can evidently be satisfied only for a fairly smooth change in the layer thickness over the body surface, i.e., when $\partial h/\partial q_1$ and $\partial h/\partial q_2$ are small.

We will eliminate the velocity vector component v_3 from the three-dimensional equations of fluid motion in the layer by taking account of the boundary conditions for $q_3 = a$ and $a+h$. On the surface of the body, we have the usual boundary condition

$$v_3 = 0 \tag{1.3}$$

for $q_3 = a$.

From the condition that the fluid should not penetrate through the contact surface we obtain on the outer surface of the compressible layer $q_3 = a + h$

$$v_3 = \frac{\partial h}{\partial t} + \frac{v_1}{H_1}\frac{\partial h}{\partial q_1} + \frac{v_2}{H_2}\frac{\partial h}{\partial q_2} \tag{1.4}$$

Writing the equations of fluid motion in curvilinear orthogonal coordinates /2/ in divergent form, we obtain

$$\frac{\partial}{\partial t}(H_1H_2\rho) + \frac{\partial}{\partial q_1}(\rho v_1H_2) + \frac{\partial}{\partial q_2}(\rho v_2H_1) + \frac{\partial}{\partial q_3}(\rho v_3H_1H_2) = 0 \tag{1.5}$$

$$\frac{\partial}{\partial t}(H_1H_2\rho v_1) + \frac{\partial}{\partial q_1}[(\rho v_1^2 + p)H_2] + \frac{\partial}{\partial q_2}(\rho v_1v_2H_1) + \frac{\partial}{\partial q_3}(\rho v_1v_3H_1H_2) = (\rho v_2^2 + p)\frac{\partial H_2}{\partial q_1} - \rho v_1v_2\frac{\partial H_1}{\partial q_2}$$

(the equation of motion is written down only in projections on the q_1 axis). Integrating (1.5) with respect to the variable q_3 between $q_3 = a$ and $q_3 = a + h$ with the above assumptions and taking the boundary conditions (1.3) and (1.4) into account, we obtain the equations of quasi-two-dimensional flow in a layer

$$H_1H_2\frac{\partial}{\partial t}(h\rho) + \frac{\partial}{\partial q_1}(h\rho\langle v_1\rangle H_2) + \frac{\partial}{\partial q_2}(h\rho\langle v_2\rangle H_1) = 0 \tag{1.6}$$

$$H_1H_2\frac{\partial}{\partial t}(h\rho\langle v_1\rangle) + \frac{\partial}{\partial q_1}(h\rho\langle v_1\rangle^2H_2) + \frac{\partial}{\partial q_2}(h\rho\langle v_1\rangle\langle v_2\rangle H_1) + H_2h\frac{\partial p}{\partial q_1} = h\rho\langle v_2\rangle^2\frac{\partial H_2}{\partial q_1} - h\rho\langle v_1\rangle\langle v_2\rangle\frac{\partial H_1}{\partial q_2}$$

(the equation of motion in projections on the q_2 axis is obtained from the second equation in (1.6) by interchanging the subscripts 1 and 2).

Note that (1.6) for the one-dimensional case will naturally agree with the equations of quasi-one-dimensional gas flow in a tube of variable section /3/.

We will use the equations obtained as boundary conditions to solve problems of diffraction by the body under consideration. Because of the thinness of the layer covering the body, the boundary conditions refer to the surface $\mathbf{r} = \mathbf{r}(q_1, q_2, a)$.

The same curvilinear coordinates are used in the outer fluid flow as when deriving (1.6). We will assume the pressures in the outer fluid flow on the surface $\mathbf{r} = \mathbf{r}(q_1, q_2, a)$ and the layer covering the body to be equal in order to obtain a closed system of boundary relations. Moreover, the boundary condition (1.4) should be satisfied on the surface $\mathbf{r} = \mathbf{r}(q_1, q_2, a)$ in the outer flow, but should be written for the velocity in the outer fluid flow (the velocities in the layer and in the outer flow are not the same).

It can be seen that the set of boundary relations obtained is equivalent to one boundary condition connecting the outer flow parameters and their derivatives on the boundary $\mathbf{r} = \mathbf{r}(q_1, q_2, a)$. We will write this boundary condition in explicit form only in the acoustic approximation. After linearizing the boundary conditions, we have on the surface $\mathbf{r} = \mathbf{r}(q_1, q_2, a)$

$$v_3 = \partial h/\partial t \tag{1.7}$$

$$H_1H_2\left(\frac{1}{h_0}\frac{\partial h}{\partial t} + \frac{1}{\rho_*c_*^2}\frac{\partial p}{\partial t}\right) + \frac{\partial}{\partial q_1}(\langle v_1\rangle H_2) + \frac{\partial}{\partial q_2}(\langle v_2\rangle H_1) = 0$$

$$\frac{\partial\langle v_1\rangle}{\partial t} = -\frac{1}{\rho_*H_1}\frac{\partial p}{\partial q_1},\quad \frac{\partial\langle v_2\rangle}{\partial t} = -\frac{1}{\rho_*H_2}\frac{\partial p}{\partial q_2},\quad h_0 = h(q_1, q_2, 0)$$

The constants ρ_*, c_* are the density of the material and the speed of sound, respectively.

We will attach the linearized Euler equation for the outer fluid flow in projections on the q_3 axis to system (1.7) (ρ_0 is the fluid density in the outer flow)

$$\frac{\partial v_3}{\partial t} = -\frac{1}{\rho_0}\frac{\partial p}{\partial q_3} \tag{1.8}$$

Eliminating the unknowns h, $\langle v_1\rangle$, $\langle v_2\rangle$ from (1.7) and (1.8), we obtain a boundary condition for the pressure on the surface $\mathbf{r} = \mathbf{r}(q_1, q_2, a)$

$$\frac{\partial^2 p}{\partial t^2} - \frac{\rho_*c_*^2}{h_0\rho_0}\frac{\partial p}{\partial q_3} = \frac{c_*^2}{H_1H_2}\left[\frac{\partial}{\partial q_1}\left(\frac{H_2}{H_1}\frac{\partial p}{\partial q_1}\right) + \frac{\partial}{\partial q_2}\left(\frac{H_1}{H_2}\frac{\partial p}{\partial q_2}\right)\right] \tag{1.9}$$

Relation (1.9) is a boundary condition for the wave equation

$$\frac{\partial^2 p}{\partial t^2} = c_0^2 \Delta p \tag{1.10}$$

For $c_* \ll c_0$ the following condition can be used together with (1.9) /1/ :

$$\frac{\partial^2 p}{\partial t^2} - \frac{\rho_* c_*^2}{h_0 \rho_0} \frac{\partial p}{\partial q_3} = 0$$

The constant $(\rho_* c_*^2)/(h_0\rho_0)$ characterizes the compressibility of the damping layer covering the body under consideration, and has the dimensions of acceleration. Note that the same condition is obtained on a free fluid surface subjected to the force of gravity /2/.

As $(\rho_* c_*^2)/(h_0\rho_0) \to \infty$. (the quantity c_* is bounded), the condition $\partial p/\partial q_3 = 0$ is obtained on a rigid body from (1.9), and as $(\rho_* c_*^2)/(h_0\rho_0) \to 0$ $(c_* \to 0)$ the condition on an absolutely soft body with a given pressure on its boundary is obtained.

2. Diffraction by a Sphere.

In a space filled with a fluid at rest with initial pressure p_0 and density ρ_0, suppose there is a rigid fixed sphere of radius a with centre at the origin of a system of spherical coordinates r, ψ, φ $(x = r\sin\varphi\cos\psi,\ y = r\sin\varphi\sin\psi,\ z = r\cos\varphi)$, covered by a damping layer of initial thickness h_0. It can be shown that the condition $h_0 \ll a$ must be imposed to satisfy the inequalities (1.1). Consequently, as above, we refer the boundary condition on the layer-fluid contact surface to the sphere $r = a$.

From infinity, let a plane acoustic wave of pressure p_i, whose front is perpendicular to the z axis and reaches the sphere $r = a$ at the time $t = 0$, impinge on the sphere

$$p_i = p_m \exp\left[-\frac{t + (z-a)/c_0}{\theta_0}\right]\eta\left(t + \frac{z-a}{c_0}\right) + p_0 \tag{2.1}$$

$$\eta(t) = \begin{cases} 0, & t \leqslant 0 \\ 1, & t > 0 \end{cases}$$

where p_m is the pressure drop at the front θ_0 is a constant with the dimensions of time, characterizing the wave duration, and c_0 is the speed of sound in the fluid.

In the axisymmetric problem under consideration, the resultant pressure field $p(r, \varphi, t)$ is described by wave equation (1.10), where

$$p = p_i,\ t \leqslant 0 \tag{2.2}$$

In the spherical coordinate system, the boundary condition (1.9) on the surface $r = a$ has the form

$$\frac{\partial^2 p}{\partial t^2} - \frac{\rho_* c_*^2}{h_0\rho_0}\frac{\partial p}{\partial r} = \frac{c_*^2}{a^2}\left(\frac{\partial^2 p}{\partial \varphi^2} + \operatorname{ctg}\varphi \frac{\partial p}{\partial \varphi}\right) \tag{2.3}$$

Representing the total pressure p in the form

$$p = p_s + p_i \tag{2.4}$$

where p_s is a perturbation which the sphere induces in the impinging wave pressure field, and introducing the dimensionless quantities

$$\bar r = \frac{r}{a},\quad \bar t = \frac{c_0 t}{a},\quad \bar p_s = \frac{p_s}{p_m},\quad \bar p_i = \frac{p_i - p_0}{p_m},\quad \bar p = \frac{p - p_0}{p_m}$$

$$\theta = \frac{a}{c_0\theta_0},\quad \gamma = \frac{\rho_0 c_0^2 h_0}{\rho_* c_*^2 a},\quad \nu = \frac{c_*}{c_0}$$

we obtain the following set of equations for the dimensionless pressure p_s from relations (1.10),(2.1)-(2.3) (we omit the bar above the dimensionless quantities throughout)

$$\frac{\partial^2 p_s}{\partial r^2} + \frac{2}{r}\frac{\partial p_s}{\partial r} + \frac{1}{r^2}\frac{\partial^2 p_s}{\partial \varphi^2} + \frac{\operatorname{ctg}\varphi}{r^2}\frac{\partial p_s}{\partial \varphi} = \frac{\partial^2 p_s}{\partial t^2},\quad t > 0, r > 1 \tag{2.5}$$

$$\frac{\partial^2 (p_i + p_s)}{\partial r^2} + \left(2 - \frac{1}{\gamma}\right)\frac{\partial (p_i + p_s)}{\partial r} + (1 - \nu^2)\left[\frac{\partial^2 (p_i + p_s)}{\partial \varphi^2} + \operatorname{ctg}\varphi\frac{\partial (p_i + p_s)}{\partial \varphi}\right] = 0,\quad r = 1 \tag{2.6}$$

$$p_s = 0,\ t \leqslant 0$$

$$p_i = \exp\left[-\theta\,(r\cos\varphi - 1 + t)\right]\eta\,(r\cos\varphi - 1 + t)$$

Note that the derivative $\partial^2(p_i + p_s)/\partial t^2$ in the first boundary condition (2.6) is eliminated by using (2.5). Applying a Laplace transform in t to (2.5) and (2.6), we obtain

$$\frac{\partial^2 p_s^*}{\partial r^2} + \frac{2}{r}\frac{\partial p_s^*}{\partial r} + \frac{1}{r^2}\frac{\partial^2 p_s^*}{\partial \varphi^2} + \frac{\operatorname{ctg}\varphi}{r^2}\frac{\partial p_s^*}{\partial \varphi} = s^2 p_s^*,\quad r > 1 \tag{2.7}$$

$$\frac{\partial^2 p_s^*}{\partial r^2} + \left(2 - \frac{1}{\gamma}\right)\frac{\partial p_s^*}{\partial r} + (1 - \nu^2)\left[\frac{\partial^2 p_s^*}{\partial \varphi^2} + \operatorname{ctg}\varphi\frac{\partial p_s^*}{\partial \varphi}\right] = \tag{2.8}$$

$$-\frac{\partial^2 p_i^*}{\partial r^2} - \left(2 - \frac{1}{\gamma}\right)\frac{\partial p_i^*}{\partial r} - (1 - \nu^2)\left[\frac{\partial^2 p_i^*}{\partial \varphi^2} + \operatorname{ctg}\varphi\frac{\partial p_i^*}{\partial \varphi}\right],\qquad r = 1$$

$$p_s^* \to 0, \quad r \to \infty; \quad p_i^* = \frac{\exp[-s(1 - r\cos\varphi)]}{s + \vartheta}$$

$$\left(f^*(s, r, \varphi) = \int_{-\infty}^{\infty} f(t, r, \varphi) e^{-st}\, dt\right)$$

Here $\operatorname{Re} s > 0$ in (2.7) and (2.8) since $p_i = p_s \equiv 0$ for $t < 1 - r\cos\varphi$. If the solution of (2.7) is sought in the form of a series in Legendre polynomials, then, taking account of the second condition in (2.8), we find

$$p_s^* = \sum_{n=0}^{\infty} B_n \frac{K_{n+1/2}(sr)}{\sqrt{r}} P_n(\cos\varphi) \tag{2.9}$$

$$B_n \frac{K_{n+1/2}(sr)}{\sqrt{r}} = \left(n + \frac{1}{2}\right) \int_0^{\pi} p_s^* P_n(\cos\varphi) \sin\varphi\, d\varphi$$

Here $K_{n+1/2}(z)$ is the modified Bessel function of the third kind, and $P_n(x)$ is the Legendre polynomial.

The incident wave is represented in the form

$$p_i^* = \frac{e^{-s}}{s + \vartheta} \sqrt{\frac{2\pi}{sr}} \sum_{n=0}^{\infty} \left(n + \frac{1}{2}\right) I_{n+1/2}(sr) P_n(\cos\varphi) \tag{2.10}$$

where $I_{n+1/2}(z)$ is the modified Bessel function of the first kind.

To determine B_n we use the boundary condition. Then by using (2.9) and (2.10), we finally obtain the following expression for B_n from the first condition in (2.8):

$$B_n = -\frac{e^{-s}}{s + \vartheta} \sqrt{\frac{2\pi}{s}} \left(n + \frac{1}{2}\right) \frac{2sI'_{n+1/2}(s) - (1 + 2\gamma\alpha_n) I_{n+1/2}(s)}{2sK'_{n+1/2}(s) - (1 + 2\gamma\alpha_n) K_{n+1/2}(s)} \tag{2.11}$$

$$\alpha_n = s^2 + \nu^2 n(n+1)$$

The expression $p^*(1, \varphi, s)$ for the transform of the pressure on the sphere surface can be represented in the form

$$p^*(1, \varphi, s) = \frac{e^{-s}}{s + \vartheta} \sqrt{\frac{2\pi}{s}} \sum_{n=0}^{\infty} \frac{\left(n + \frac{1}{2}\right) P_n(\cos\varphi)}{\gamma\alpha_n K_{n+1/2}(s) - sK'_{n+1/2}(s) + \frac{1}{2} K_{n+1/2}(s)} \tag{2.12}$$

By passing to the limit as $\gamma \to 0, \infty$ in (2.12) we obtain the well-known solution of acoustic-wave diffraction problems by, respectively, an absolutely rigid sphere, and an absolutely soft sphere ($p = 0$ for $r = 1$) /4/.

When the parameter γ has finite values it is convenient to express it in the form $\gamma = \varepsilon/(\mu\nu)$, where $\varepsilon = h_0/a$ is the dimensionless thickness of the damping layer, and $\mu = (\rho_* c_*)/(\rho_0 c_0)$ is the ratio of the acoustic impedances of the damping layer and the surrounding fluid.

Let us examine the same diffraction problem as above, but without the assumption that the pressure is constant in the layer along the radius by considering this quantity to satisfy the wave equation both outside and inside the damping layer (the speeds of sound in these regions are different). It is here more convenient to change the scale along the r axis so that the outer boundary of the damping layer has the equation $r = 1$, and the surface of the solid sphere has the equation $r = 1 - \varepsilon$. Then the first boundary condition of (2.6) is replaced by the conditions for the pressures and the normal velocity components on the outer boundary of the damping layer to be equal, while a condition that the normal derivative of the pressure is zero appears on the sphere surface (the impenetrability condition of a rigid body).

The Laplace transform $p_{s,w}^*$ of the solution of the problem has the following form for $r = 1$:

$$p_{s,w}^* = \sum_{n=0}^{\infty} C_n(s) K_{n+1/2}(s) P_n(\cos\varphi) \tag{2.13}$$

$$C_n(s) = -\frac{e^{-s}}{s + \vartheta} \sqrt{\frac{2\pi}{s}} \left(n + \frac{1}{2}\right) \frac{2sI'_{n+1/2}(s) - I_{n+1/2}(s)[1 + D_n(z_1, z)]}{2sK'_{n+1/2}(s) - K_{n+1/2}(s)[1 + D_n(z_1, z)]}$$

$$D_n(z_1, z) = \frac{\nu}{\mu} \frac{u_n(z) v_n(z_1) - u_n(z_1) v_n(z)}{K_{n+1/2}(z) u_n(z_1) - I_{n+1/2}(z) v_n(z_1)}$$

$$u_n(z) = I_{n+1/2}(z) - 2zI'_{n+1/2}(z), \quad v_n(z) = K_{n+1/2}(z) - 2zK'_{n+1/2}(z)$$

$$z = s/\nu, \quad z_1 = (1 - \varepsilon) z$$

We shall henceforth consider ε and μ to be small quantities, but such that the quantity γ is finite. Expanding (2.13) in powers of ε to the second order inclusive, we obtain

$$p_{s,w}^* \approx p_s^* + \varepsilon R \tag{2.14}$$

$$R = \frac{8\gamma s^2 e^{-s}}{s+\vartheta} \sum_{n=0}^{\infty} \left(n + \frac{1}{2}\right) \frac{K_{n+1/2}(s) P_n(\cos\varphi)}{[2sK'_{n+1/2}(s) - (1 + 2\gamma\alpha_n) K_{n+1/2}(s)]^2}$$

where $p_s{}^*$ is given by (2.9) and (2.11) for $r = 1$. Carrying out an inverse Laplace transform in (2.14), we obtain, since $R(s)$ has an original, that the solution, taking into account the wave in the layer, differs very little from the solution when the wave is ignored for small ε. However, the solution ignoring the wave is continuous, unlike the solution taking the wave into account, and also, being fairly simple, it is very promising for analyzing its properties.

3. A Force Acting on a Sphere. The transform of the dimensionless force F acting on a sphere can be obtained from (2.9)-(2.12) by using the orthogonality of the Legendre polynomials and the relationship $K_{3/2} = \sqrt{\pi/(2s)}\, e^{-s}(s + 1)/s$ $(\bar{F} = F/(\pi a^2 p_m))$

$$F^* = 2\int_0^{\pi} (p_i{}^* + p_s{}^*) \cos\varphi \sin\varphi \, d\varphi = \frac{4s}{s+\vartheta} \frac{1}{\Delta(s)} \tag{3.1}$$

$$\Delta(s) = \gamma s^3 + (\gamma + 1) s^2 + 2(1 + g)(s + 1), \quad g = \gamma\nu^2 = h_0\rho_0/(a\rho_*)$$

The bar above the F is omitted here and henceforth. Applying the inverse Laplace transform to (3.1), we obtain an expression for the force acting on a sphere

$$F = -\frac{4\vartheta e^{-\vartheta t}}{\Delta(-\vartheta)} + \sum_{k=0}^{2} \frac{4s_k e^{s_k t}}{(s_k + \vartheta)[3\gamma s_k^2 + 2(\gamma + 1)s_k + 2(1 + g)]} \tag{3.2}$$

Here s_k $(k = 0, 1, 2)$ are the roots of the cubic equation $\Delta(s) = 0$ (generally complex).

Rejection of the assumption of sphere immobility does not result in any appreciable complication of the problem. A term $(-\gamma^{-1}\cos\varphi\, dV/dt)$ is added on the right side of boundary condition (2.6), and moreover, the following additional condition appears:

$$\frac{2}{3} m \frac{dV}{dt} = \int_0^{\pi} (p_i + p_s) \cos\varphi \sin\varphi d\varphi, \quad m = \frac{3M}{4\pi a^3 \rho_0}$$

(M and V are the mass and velocity of the sphere respectively). These changes result in the same formula for the quantity F as in (3.2), however, the term $(2m)^{-1}$ is here attached to the coefficient g.

The total momentum communicated to the sphere during diffraction turns out to be dependent on their elastic properties of the damper

$$I = \int_0^{\infty} F\, dt = \begin{cases} 0, & \vartheta \neq 0 \\ \left(1 + \frac{h_0\rho_0}{a\rho_*} + \frac{1}{2m}\right)^{-1}, & \vartheta = 0 \end{cases}$$

as can be shown using (3.1).

As an illustration, the properties of expression (3.2) are represented in Figs.1 and 2 by time dependences of the magnitude of the force acting on the sphere for different values of the parameter γ and g when a step wave is incident $(\vartheta = 0)$.

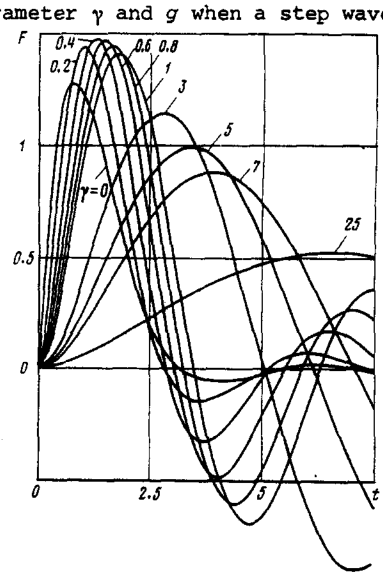

Fig.1

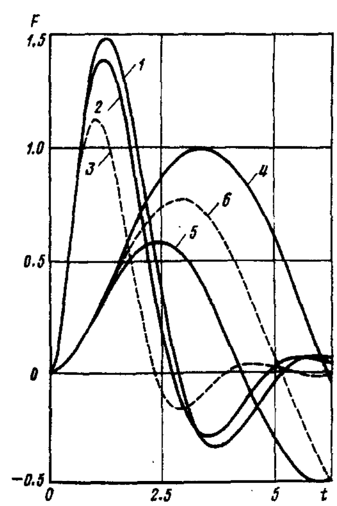

Fig.2

Graphs of the function $F(t)$ are constructed in Fig.1 assuming $g = 0$, and values of γ corresponding to the curves are also indicated.

The non-monotonic dependence of the maximum value of F on the parameter γ characterizing the elastic properties of the damper is interesting: as γ increases, this value first increases, thereby turning out to be higher than the value of the maximum force acting on a rigid sphere, and only then decreasing to become less. The effect of exceeding the maximum force acting on a sphere covered by a damper as compared with the maximum force acting on a rigid sphere is caused by the three-dimensionality of the flow that occurs: it is missing in the one-dimensional case /1/. The range of values of γ corresponding to the effect described corresponds to a very thin damper for real media $(h_0/a \sim 10^{-4})$, hence it can also be observed without the presence of a damping layer, in particular, because of gas bubbles in the fluid near the sphere, or for other reasons.

The graphs of the function $F(t)$ constructed in Fig.2 correspond to two fixed values of γ: 0.4 (curves *1,2,3)* and 5 (curves *4,5,6)* and different values of g: the values $m = 0, \nu = 0$ correspond to curves *1* and *4*; $m = 0$, $\nu = 0.5$ to curves *2* and *5*, and the dashed curves *3* and *6* are constructed to illustrate the dependence $F(t)$ for a sphere of finite mass for $m = 1$, $\nu = 0$.

4. Approximate computation of the pressure.

Since applying the inverse Laplace transform involves considerable difficulties even for the first few terms in series (2.12), it is better to use an approximate approach as in piston theory/4, 5/, say, which enables a simple formula to be obtained for the pressure distribution in the frontal part of the sphere. To do this, the derivatives with respect to φ should be omitted in (2.5) and (2.6). The solution of the appropriate problem results in the following time dependence of the pressure on the sphere surface:

$$p = \left[1 - \left(\operatorname{ch}\beta t_1 - \frac{1+2\cos\varphi}{\beta}\operatorname{sh}\beta t_1\right)e^{-t_1}\right]\eta(t_1), \qquad t_1 = \frac{t-1+\cos\varphi}{2\gamma}, \qquad \beta = \begin{cases}\sqrt{1-4\gamma}, & \gamma \leqslant \frac{1}{4}\\ i\sqrt{4\gamma-1}, & \gamma > \frac{1}{4}\end{cases} \tag{4.1}$$

Based on (4.1), the time dependences of the pressure at the stagnation point $(\varphi = 0)$ are displayed by the solid lines 1 - 5 in Fig.3 for values of γ equal to 0, 0.05, 0.25, 0.4, 1. The maximum that always exists (for $\gamma \neq 0, \infty$) is reached at a time $t = (2\gamma/\beta)\operatorname{Arth}[\beta/(1-\gamma)]$, and will be greater, the greater the value of γ. The formula

$$p = 2(1 - e^{-t/\gamma}) \tag{4.2}$$

is obtained in /1/ for the pressure behind the reflected wave in the case of the one-dimensional problem of wave incidence on a wall covered by a damper. It is natural to expect agreement between the pressures obtained by means of (4.1) and (4.2) at the initial times. They can be compared in Fig.3, where the dashed lines correspond to reflection from a flat wall.

Piston theory may not be sufficiently accurate for times of the order of one. However, the accuracy of (4.1) can be clarified by comparing the time dependence of the force obtained from it and, for brevity, is not here referred to the exact dependence (3.2). This comparison shows that they agree satisfactorily in the range $0 < t < 2$ of practical interest, when the incident wave front intersects the sphere surface /1/.

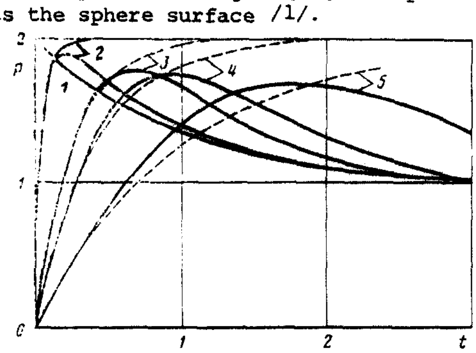

Fig.3

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